Landau-Zener transitions in a superconducting flux qubit

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We report an experimental measurement of Landau-Zener transitions on an individual flux qubit within a multiqubit superconducting chip. The method used isolates a single qubit, tunes its tunneling amplitude Δ into the limit where Δ is much less than both the temperature T and the decoherence-induced energy level broadening, and forces it to undergo a Landau-Zener transition. We find that the behavior of the qubit agrees to a high degree of accuracy with theoretical predictions for Landau-Zener transition probabilities for a double-well quantum system coupled to a nonMarkovian $1/f$ magnetic flux noise.

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Adiabatic quantum computation (AQC) is a quantum mechanical method for solving hard computational problems.[1] In AQC one encodes a computational problem in a suitable physical system. If one can somehow place the system in its ground or lowest-energy state, the structure of that ground state then reveals the answer to the problem. To find that ground state using AQC, one starts by engineering a simple Hamiltonian or energy functional for the system and by placing the system in the ground state of this simple Hamiltonian. Then one gradually deforms the Hamiltonian of the system from the simple form into a complex Hamiltonian whose ground state encodes the answer to the problem. If this deformation is sufficiently gradual, then the transformation of the state of the system is adiabatic, and the system remains in its ground state throughout the deformation. AQC is known to be a universal model of quantum computation.[2]

A key question of AQC is whether adiabaticity can be maintained throughout the computation. The Landau-Zener (LZ) transitions may take the system out of its ground state, unless the time over which one transverses the point of the minimum energy gap during the course of a computation is longer than the instantaneous coupling between ground and excited states divided by the minimum gap squared. This is the case for both isolated systems[3,4] and systems with dissipation and decoherence.[5] Just which hard problems can be encoded in such a way so that adiabaticity can be maintained over reasonable times remains an open question. Moreover, for some optimization problems, an excited final state with sufficiently low energy could provide an acceptable solution and therefore the adiabaticity condition may be relaxed.

Equally important question is whether interactions between the computer and its environment can spoil the computation. Unlike in the gate model of quantum computation,[6] the effects of an environment on AQC are less understood. While it is now clear that AQC has fundamental advantages over the gate model in regards to robustness against decoherence,[7–9] there does not yet exist an equivalent of the threshold theorem[6] although fault tolerant schemes for AQC have been proposed.[10] Nonetheless, a clear understanding of how LZ transitions are affected by an environment represents an important step forward.

For large-scale hard problems, it is inevitable that any implementation of AQC will encounter minimum gaps that are smaller than temperature T and decoherence rate. In order to understand the effects of environment in this limit, we attempt to first understand in detail how environment affects LZ transitions in individual qubits. We purposely operate in a regime in which the decoherence time scale τ_φ is much shorter than the adiabatic passage, and in which $T \gg \Delta$, where Δ is the tunneling amplitude. While evidence of LZ transitions has been reported before in molecular nanomagnets[11] as well as superconducting qubits,[12–14] this Brief Report reports, *direct* measurement of LZ transitions in a superconducting flux qubit in the high T and strong decoherence regime.

In the original LZ problem, the system Hamiltonian is

$$H_S = -(\Delta\sigma_x + \epsilon\sigma_z)/2, \tag{1}$$

with $\epsilon = \nu t$, where $\sigma_{x,z}$ are the Pauli matrices and ν is the sweep rate for the energy bias. We take $|0\rangle$ and $|1\rangle$ to be eigenfunctions of σ_z, denoting the "left" and "right" states in a double-well potential which can represent the two flux states in a superconducting flux qubit. If at $t=-\infty$ the system starts in state $|0\rangle$, then the probability of finding it in the same state at time $t=+\infty$ is *exactly* given by[3,4]

$$P_{\rm LZ} = e^{-\pi\Delta^2/2\nu}. \tag{2}$$

If Hamiltonian (1) describes the dynamics of the two lowest energy states in a multiqubit adiabatic quantum computer close to the energy anticrossing, then Eq. (2) has the probability of failing to reach the final ground state in the decoherence-free system. Now suppose that the qubit is coupled to an environment. The total Hamiltonian $H=H_S+H_B+H_{\rm int}$ comprises system (1) and environment H_B parts, and an interaction Hamiltonian

$$H_{\rm int} = -Q\sigma_z/2 \tag{3}$$

that provides coupling between the qubits, and an operator Q that acts on the environment. Here, we only consider longitudinal coupling to the environment, which represents flux noise affecting the flux bias in a flux qubit. We do not specify H_B explicitly because if environmental fluctuations obey Gaussian statistics then all averages can be expressed in terms of the spectral density $S(\omega)=\int_{-\infty}^{\infty} dt e^{i\omega t}\langle Q(t)Q(0)\rangle$.

Here, $\langle \ldots \rangle$ denotes the averaging over environmental degrees of freedom. Hamiltonian (3) is what one expects for the effective interaction Hamiltonian for a large-scale AQC at the anticrossing, regardless of the type of coupling of individual qubits to the environment.[9]

An immediate consequence of coupling to the environment is that the relative phase between the two terms in the wave function that correspond to the two energy levels becomes uncertain after some time; an effect known as dephasing or decoherence. Due to energy time uncertainty, the pure dephasing time τ_φ is inversely related to the uncertainty in the energy eigenvalues or so called broadening W of the energy levels: $\tau_\varphi \propto 1/W$. If $W, T \ll \Delta$, then the system will have a well-defined ground state separated from the excited state by a well-defined gap and thermal transitions will be suppressed. One would then expect that Eq. (2) holds even in the presence of noise although Δ may be renormalized by high-frequency modes of the environment.[15,16] The important question now is what happens when $W, T \gg \Delta$ so that the broadened ground and first excited states merge into each other and thermal transitions completely mix them up. Here, we answer this question both theoretically and experimentally.

In the regime $\Delta \ll W$, the dynamics of the system becomes incoherent. Using second-order perturbation in Δ and assuming that the environment is dominated by low-frequency Gaussian noise, the incoherent tunneling rate from $|0\rangle$ to $|1\rangle$ is given by[17]

$$\Gamma_{01}(\epsilon) = \sqrt{\frac{\pi}{8}}\frac{\Delta^2}{W}\exp\left[-\frac{(\epsilon-\epsilon_p)^2}{2W^2}\right], \tag{4}$$

$$W^2 = \int \frac{d\omega}{2\pi}S(\omega), \quad \epsilon_p = \mathcal{P}\int \frac{d\omega}{2\pi}\frac{S(\omega)}{\omega}, \tag{5}$$

with backward transition given by $\Gamma_{10}(\epsilon)=\Gamma_{01}(-\epsilon)$. The transition rates therefore exhibit a Gaussian peak with a center shifted away from the resonance point $\epsilon=0$. The width of the transition region, W, which is a measure of the environmentally induced broadening of the energy levels, is thus given by the rms value of the noise. In thermal equilibrium, the width W and the position ϵ_p of such macroscopic resonant tunneling (MRT) peaks are related by[17]

$$W^2 = 2T\epsilon_p. \tag{6}$$

These predictions have been experimentally confirmed using superconducting flux qubits.[18] Let us now return to the LZ problem. Suppose at $t=t_i$ the system starts from $|0\rangle$ with probability $P_0(t_i)=1$. In the incoherent tunneling regime $(\Delta \ll W)$, the off-diagonal elements of the density matrix vanish much faster than the evolution of the diagonal elements (within timescale $\tau_\varphi \sim 1/W \ll 1/\Gamma_{01,10}$). Thus to find $P_0(t)$, one needs to solve the equation[19]

$$\dot{P}_0 = -\Gamma_{01}P_0 + \Gamma_{10}(1-P_0). \tag{7}$$

In the low-temperature regime $T \ll W$ (but can be $\gg \Delta$), Eq. (6) requires that $W \ll 2\epsilon_p$, thus separating the peaks of Γ_{01} and Γ_{10} such that $\Gamma_{01}(-\epsilon_p) \ll \Gamma_{01}(\epsilon_p)$. One can therefore neglect the second term in Eq. (7); because at points where

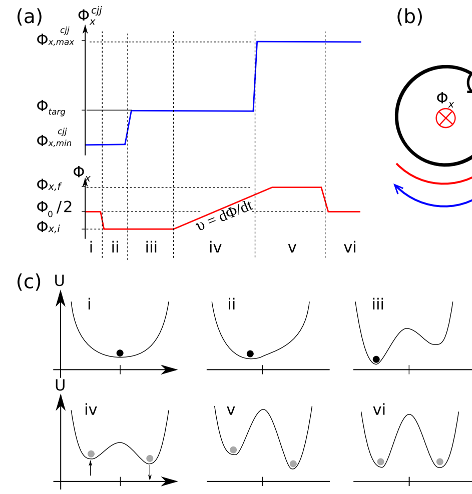

FIG. 1. (Color online) Schematics of (a) the pulse sequence for the LZ measurements, (b) a CJJ rf-SQUID qubit, and (c) the qubit potential during the measurement.

Γ_{10} is peaked, $1-P_0 \approx 0$, and at all other points, $\Gamma_{10} \approx 0$. The probability P_0 will then be approximately given by

$$P_0(t_f) = \exp\left[-\int_{t_i}^{t_f}\Gamma_{01}(\tau)d\tau\right] = e^{-(\pi\Delta^2/2\nu)\kappa}, \tag{8}$$

$$\kappa = \frac{1}{\sqrt{2\pi}W}\int_{\epsilon_i}^{\epsilon_f}\exp\left[-\frac{(\epsilon-\epsilon_p)^2}{2W^2}\right]d\epsilon. \tag{9}$$

For $\epsilon_i \to -\infty$, this equation becomes

$$\kappa = \frac{1}{2}\left[1+\mathrm{erf}\left(\frac{\epsilon_f-\epsilon_p}{\sqrt{2}W}\right)\right]. \tag{10}$$

If also $\epsilon_f \to \infty$, then $\kappa=1$, yielding (2), which is exactly the LZ transition probability in a completely coherent system, in agreement with previous studies of dissipative Landau-Zener transitions.[5] If the condition $W \gg T$ does not hold, then one must keep all of the terms in Eq. (7) and calculate P_0 numerically.

We have experimentally tested the above predictions by examining LZ transitions using a single decoupled qubit in a 28 qubit chip designed for adiabatic quantum computation. The sample was cooled down in a magnetically shielded dilution refrigerator with heavily filtered lines to a base temperature of about 10 mK. The qubits on the chip were compound Josephson-junction (CJJ) rf-superconducting quantum interference device (SQUID) qubits as schematically shown in Fig. 1(b) and described in Ref. 18. Each qubit consists of a main loop and a CJJ loop subjected to external flux biases Φ_x and Φ_x^{cjj}, respectively. The CJJ loop is interrupted by two nominally identical Josephson junctions connected in parallel. This device can be operated as a qubit for $\Phi_x^{\mathrm{cjj}} \in [0.5,1]\Phi_0$ and $\Phi_x \approx 0$, where Φ_0 is the flux quantum. The two oppositely circulating persistent current states correspond to the states $|0\rangle$ and $|1\rangle$. The bias energy is $\epsilon = 2|I_p|\Phi_x$, where the I_p is the persistent current. The parameter Δ is the amplitude of the flux tunneling between the two

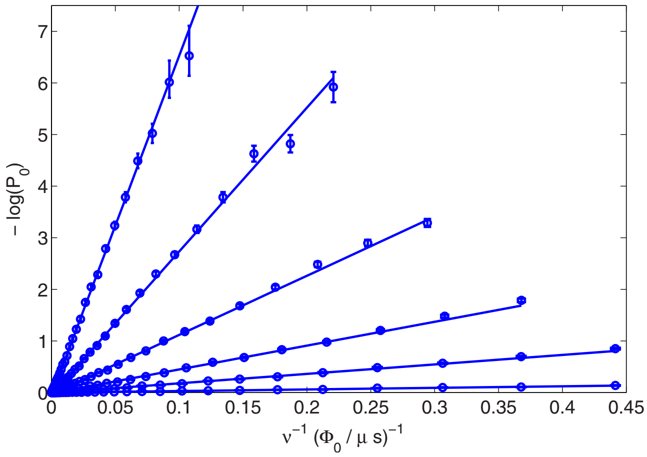

FIG. 2. (Color online) LZ probability as a function of inverse sweep rate for different values of Δ. The lines are linear fits to the data. $\Phi_x^{\mathrm{cjj}}/\Phi_0=-0.7395$, -0.742, -0.7445, -0.747, -0.7495, and -0.752 from top to bottom.

states. Both $|I_p|$ and Δ are controlled by Φ_x^{cjj}. Maximum $\Delta \sim \omega_p \sim 20$ GHz, where ω_p is the plasma frequency of the rf SQUID, is obtained at $\Phi_x^{\mathrm{cjj}}=\Phi_0/2$. For $\Phi_x^{\mathrm{cjj}} \approx \Phi_0$, $\Delta \to 0$ and the system becomes localized in $|0\rangle$ or $|1\rangle$. One can then read out the qubit by measuring the flux via an inductively coupled dc SQUID (not shown).

We isolated one of the qubits by tuning the coupling between that qubit and its neighboring qubits to zero, which allowed us to perform single qubit LZ measurement. The LZ measurement is performed using the pulse sequence shown in Fig. 1(a): the qubit is first initialized in one of the states $|0\rangle$ or $|1\rangle$ at a bias $\Phi_x=\Phi_{x,i}$ [Fig. 1(a) regions i to iii] and then the bias is linearly swept from $\Phi_{x,i}$ to a final value $\Phi_{x,f}$ (regions iv and v), at which point the qubit is measured (region vi). A cartoon of the qubit potential during the pulse sequence is shown in Fig. 1(c). The probability $P_0(t_f)$ of finding the qubit in the same state $|0\rangle$ as it started from was measured by repeating the above process 2048 times for each value of the sweep rate ν. Figure 2 shows the probability

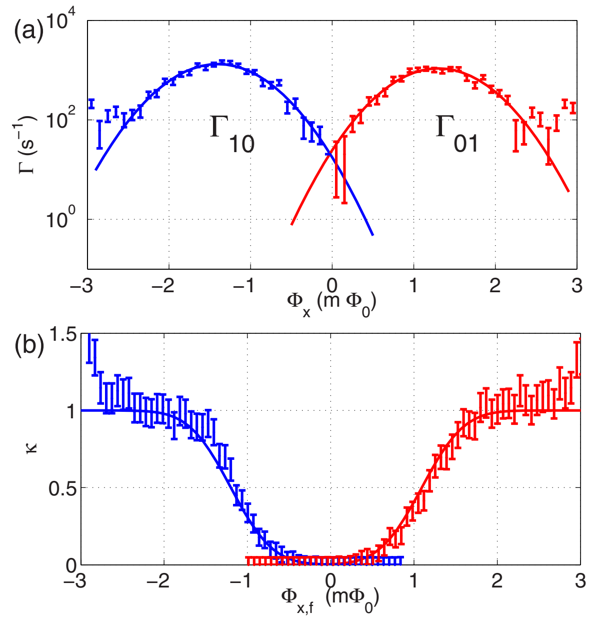

FIG. 3. (Color online) (a) First MRT peaks in Γ_{01} and Γ_{10} and their best fits with Eq. (4). (b) The experimental value of κ as defined in Eq. (8) as a function of the final bias ϵ_f. The solid line shows the theoretical curve Eq. (10) using the parameters obtained from the best fit in (a). Deviations from the fits at $|\Phi_{x,f}| > 2.5$ mΦ_0 are a result of tunneling to the second level in the target well.

$P_0(t_f)$ in logarithmic scale as a function of $1/\nu$. The result shows exponential dependence upon $1/\nu$, in agreement with Eq. (8).

Next we experimentally verify Eq. (10). We first determine Δ, W, and ϵ_p by measuring Γ_{01} and Γ_{10}, as described in Ref. 18. Figure 3(a) shows example plots of Γ_{01} and Γ_{10} as a function of bias Φ_x for the above qubit at $\Phi_x^{\mathrm{cjj}}=-0.749\Phi_0$. The line shape of the resonant peak fits very well with the Gaussian function (4), providing Δ, W, and ϵ_p as fitting parameters. The deviation from the Gaussian fit at the tails ($|\Phi_x| > 2.5m\Phi_0$) is due to transition to the second energy

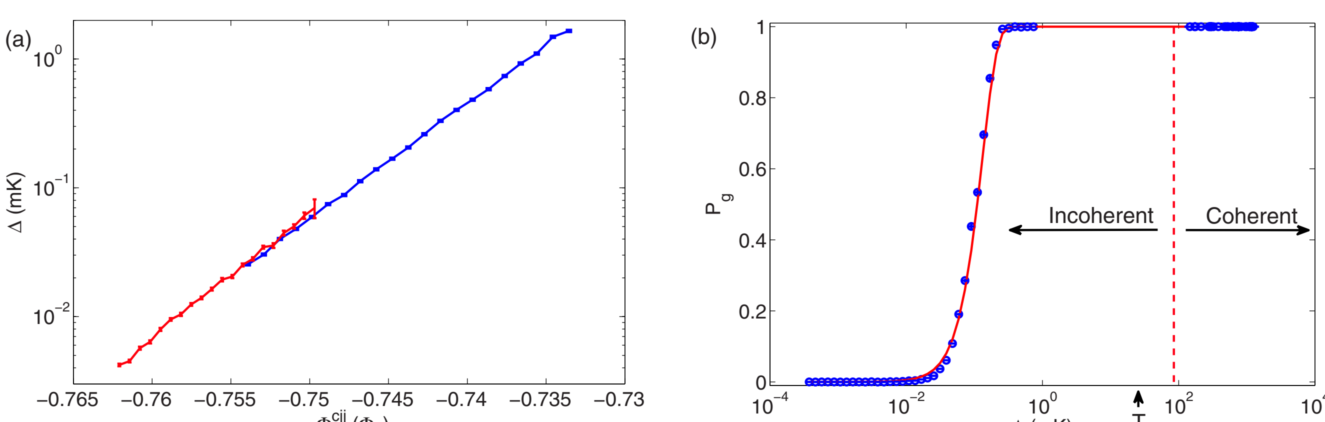

FIG. 4. (Color online) (a) Plot of Δ vs flux bias Φ_x^{cjj} applied to the compound junction. The lower curve is Δ measured with MRT and the upper curve is Δ measured with LZ. In the region where the two curves overlap they agree within the error bars. (b) Experimental (dots) and theoretical (line) ground-state probabilities $P_g=1-P_{\mathrm{LZ}}$ for fixed $\nu=0.05\Phi_0/\mu s$ as a function of Δ. The dashed line indicates the crossover between the incoherent and coherent regime defined by $W \approx \Delta$.

level in the target well.[18] For the data shown in Fig. 3(a), we found $\Delta = 0.082 \pm 0.002$ mK, $W = 123 \pm 2$ mK, and $\epsilon_p = 354 \pm 3$ mK. Equation (6) then gives the effective temperature of the sample to be $T = 21 \pm 1$ mK. Notice that the condition $T \ll W \ll 2\epsilon_p$ is approximately satisfied and therefore Eq. (8) should be sufficient to describe the LZ probability.

The transition probability as a function of flux bias was then measured for the same CJJ setting as in Fig. 3(a). Figure 3(b) shows $\kappa = -\ln P_0/(\pi\Delta^2/2\nu)$ as a function of $\Phi_{x,f}$ using the extracted Δ. The data start from zero where $\Phi_{x,f} \approx \Phi_{x,i}$ and show a plateau at $\kappa \approx 1$, in agreement with the theory. We have also plotted, on the same graph, the theoretical prediction of Eq. (10), using ϵ_p and W extracted from the MRT in Fig. 3(a), and found very good agreement with the experiment with no extra fitting parameters.

The measurements of the transition rates and the LZ probability allow us to extract Δ as a function $\Phi_x^{\rm cjj}$ for a large range of Δ.[20] Exponential dependence on $\Phi_x^{\rm cjj}$ is evident in Fig. 4(a). Figure 4(b) plots the LZ probability, for a fixed value of ν, as a function of Δ for a quite wide range of Δ (from 27 μK to 1.25 K) together with the theoretical prediction.[21] In the figure we have identified a line $\Delta = W$, which separates coherent from incoherent tunneling regime. Excellent agreement with theory is observed.

We have reported on an experimental probe of the dynamics of a flux qubit in the practically interesting regime for adiabatic quantum computation, where the energy gap Δ is much smaller than both the decoherence-induced energy level broadening W and temperature T. We find that the transition probability for the qubit quantitatively agrees with the theoretical predictions. In particular, we demonstrate that in this large decoherence limit, the quantum mechanical behavior of this qubit is the same (except for possible renormalization of Δ) as that of a noise-free qubit, as long as the energy bias sweep covers the entire region of broadening W. The close agreement between theory and experiment for a single qubit undergoing a LZ transition in the presence of noise supports the accuracy of our dynamic models, including both the noise model and the model of a single superconducting qubit. Future experiments will test the behavior of multiple coupled qubits undergoing a LZ transition in the presence of noise.

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[19] Although the slow evolution of the diagonal elements of the density matrix makes Eq. (7) local in time, the treatment is not Markovian as the decay of the off-diagonal elements of the density matrix is much faster than the response of the environment.
[20] The range is limited from below and above by longest practical measurement time and bandwidth of the measurement lines, respectively.
[21] Δ in the coherent regime was obtained by measuring the average flux and using $\langle\Phi\rangle \propto \langle\sigma_z\rangle = \epsilon/\sqrt{\epsilon^2+\Delta^2}$, which is valid in the large gap regime.